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A CHRONOLOGY AND HISTORICAL ANALYSIS OF THE MATHEMATICAL MANUSCRIPTS OF GREGORIUS A SANCTO VINCENTIO (1584-1667)

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SUMMARIES

For a long time it was believed that Gregorius a Sancto Vincentio, a Flemish Jesuit, played a secondary role in the history of mathematics. During the last fifty years this situation has changed: many significant studies about his life and works have assigned him a more important role. However, his mathematical manuscripts have remained nearly untouched. Probably many investigators have been discouraged by the mass of disorganized manuscripts, bound together in volumes with no regard to chronology or subject classification. This article contains a summary of the contents of the manuscripts, the reconstruction of their chronology, and a study of the evolution of Sancto Vincentio's mathematical ideas. In this way some discoveries attributed to Sancto Vincentio may be verified.

Le jésuite flamand Grégoire de Saint-Vincent a été considéré longtemps comme une figure secondaire dans l'histoire des mathématiques. Cette opinion a changé depuis quelques décades, car plusieurs études fouillées, concernant son oeuvre éditée, lui ont attribué un rôle plus important. Pourtant ses manuscrits mathématiques n'étaient pas encore étudiés. Les historiens des mathématiques ont probablement été découragés par la quantité des manuscrits, reliés en volumes sans tenir compte de l'ordre chronologique de leur rédaction et de la classification des matières traitées. Cet article donne un bref aperçu du contenu de ces manuscrits, de leur ordre chronologique et de l'évolution des idées mathématiques de G. de Saint-Vincent, confirmant ainsi certaines priorités qui lui sont dues.

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A CONCISE CHRONOLOGY OF THE LIFE OF SANCTO VINCENTIO

Gregorius a Sancto Vincentio was born in Bruges on September 8, 1584, but virtually nothing is known about his origins, for he never wrote about his parents. He went to the secondary school in Bruges, studied philosophy in Douai, and entered the Society of Jesus in Rome on October 21, 1605. After two years of novitiate he went to the Collegium Romanum to continue his studies. By then the famous mathematician Christophorus Clavius (1538-1612) no longer held the chair of mathematics, but it is very likely that Sancto Vincentio studied mathematics under him. However, it is known from the Elogium of Sancto Vincentio that Clavius appreciated Sancto Vincentio's talents for mathematics.

In 1612 Sancto Vincentio was sent to Louvain to finish his theological studies, and on March 23, 1613, he was ordained as a priest in Louvain. He taught Greek at the college of Brussels and for one year he was a duty-master at the college of Den Bosch. After his third year of probation in Kortrijk he became the companion of Franciscus de Aguilon (1566-1617) in the home of the Jesuits in Antwerp. With de Aguilon he organized and for a time taught a course in mathematics which the Society had, some years earlier, decided to add to the curriculum of her members. He taught mathematics from 1617 to 1620 in Antwerp. Among his pupils were Jan-Karel della Faille, Philippus Nutius Sr., Ignatius Derkennis, Antonius Alegambe, and Jacobus Durand. He continued his teaching in Louvain from 1621 to 1625, and among his pupils were Guilielmus Boelmans, Theodorus Moretus, and Joannes Ciermans. This most creative period of his life was characterized by important mathematical discoveries.

In 1625 Sancto Vincentio believed that he could solve the famous problem of the squaring of the circle, and he requested permission from the General, Mutius Vitelleschi (1563-1645), to publish a work about it. Vitelleschi, who was not a mathematician, was very cautious and advised Sancto Vincentio to send his mathematical discoveries to Christophorus Grienberger (1564-1636), the successor of Clavius at the Collegium Romanum. In 1625 Sancto Vincentio, with the help of a few collaborators (among others Guilielmus Boelmans, Theodorus Moretus, and Ignatius Derkennis), wrote several treatises to convince Grienberger that he had succeeded in squaring the circle.

The treatises did not convince Grienberger completely, and on September 9, 1625, Sancto Vincentio was allowed to leave for Rome to discuss the problem with Grienberger. In 1627 Grienberger wrote a letter to Vitelleschi in which he said that the writings of Sancto Vincentio did contain the first steps of a solution to the problem, but that for the moment the ideas were not sufficiently developed to lead to an acceptable result.

At the end of 1627 Sancto Vincentio returned to the Netherlands without having accomplished anything. In 1628 he was sent to Prague on the demand of the emperor Ferninand II. According to the Elogium he was a confessor at the imperial residence. There he was stricken by an attack of apoplexy thus ending the period of his great mathematical creativity, as is evident from the study of his manuscripts. He requested a collaborator to continue his work, and in 1629 Theodorus Moretus came to Prague to order and to rewrite the manuscripts of Sancto Vincentio.

In 1631 the Swedes invaded Prague, which became the scene of plundering and many fires, during which some of the manuscripts of Sancto Vincentio were lost. His colleague Rodericus de Arriaga (1592-1667) was able to save a number of the manuscripts and transported them to Vienna. They remained there until they were returned to Sancto Vincentio in 1641.

In 1632 Sancto Vincentio arrived at the Jesuit school in Ghent, where he stayed until he died. In the Catalogus Personarum of 1656 of the Society of Jesus the function of Sancto Vincentio in Ghent is described: "Scriptor ac professor matheseos domesticus." From the word *domesticus* we may conclude that he was a private teacher for the members of the Society. Among his pupils were Maximilianus Le Dent, Franciscus Xaverius Aynscom, Aegidius Gottignies, and Joachim Van Paepenbroek. After 1641 he was able to continue his great work on the squaring of the circle. His book, *Problema Austriacum plus ultra quadratura circuli, Auctore P. Gregorio a Sancto Vincentio Soc. Iesu, Antverpiae, apud Ioannem et Iacobum Meursios, anno M.DC. XLVII*, was published in 1647. The words *quadratura circuli* attracted the attention of mathematicians, with the result that the many beautiful discoveries contained in this treatise were largely ignored, while its readers argued whether or not the squaring of the circle had actually succeeded. The defenders of Sancto Vincentio included A. A. de Sarasa (1618-1667), G. A. Kinner de Löwenthurn (c. 1610-?), and F. X. Aynscom (1620-1660); the opponents were M. Mersenne (1588-1648), Christian Huygens (1629-1695), A. Sylvius (?-?), V. Leotaud (1596-1672), A. Auzout (1622-1691), and M. Meibom (1621-1670). Apparently Sancto Vincentio did not take part in the discussion openly; his pupils and colleagues were responsible for publications to support the squaring of the circle by Sancto Vincentio. It will, however, become evident from the discussion of the manuscripts given below that Sancto Vincentio manipulated the situation from behind the scenes.

When the controversy over the *Problema Austriacum* had subsided, Sancto Vincentio applied himself to the solution of a second classical problem; the duplication of the cube. In 1659 he was stricken by a second attack of apoplexy and on January 27, 1667, by a third, after which he died. He did not live to see the publication of his second work. A. A. de Sarasa completed the manuscripts and his last pupil, Joachim Van Paepenbroek, supervised publication of the treatise *R.P. Gregorii a Sto Vincentio Societate Iesu Opus Geometricum Posthumum ad Mesolabium per rationum proportionalium novas proprietates. Finem operis mors authoris antevertit. Gandavi, Typis Manilii, Typographi Iurati, sub signo albae columbae, Anno 1668.*

DESCRIPTION, CLASSIFICATION, AND CHRONOLOGY OF THE MANUSCRIPTS

The collection of the manuscripts left by Sancto Vincentio can be found in the Royal Library Albert I in Brussels, Department of Manuscripts, under the numbers 5770-5793. After the death of Sancto Vincentio the manuscripts were bound without regard to themes or chronology in 17 volumes. The sheets are approximately 30 by 20 cm, and the number of sheets per volume varies from 319 to 583. The original, discontinuous numbering was black and was done by Sancto Vincentio and his pupils. After binding, probably in the last century, a red numbering system was introduced for each volume. Numbers in the following catalogue refer to the red numbers paginating the manuscripts.

The manuscripts were chiefly written by Sancto Vincentio, a few by his pupils, and one by de Aguilon. All manuscripts deal with the two publications, the *Problema Austriacum* (1647) and the *Opus Posthumum* (1668). Volumes 1, 2, 6, 10, 11, 12, 13, 14, 15, and 17 deal with the *Problema Austriacum*. Volume 8 was written after publication of the *Problema Austriacum* and contains the arguments devised by Sancto Vincentio for use by his defenders. Volumes 3, 4, 5, 7, 9, and 16 contain the preliminary work to the *Opus Posthumum*.

Proper chronology was established by a number of factors, including contents, writers, internal references noted in the manuscripts, a few explicit dates, and the watermarks.

1. The oldest manuscript is to be found in Volume 6, folios 215-315, with the title *Lemmata*. It was written by F. de Aguilon, who died in 1617.

2. Following this, in chronological order, is an extensive manuscript consisting of Volumes 13 and 17, which must be considered as a whole. Sancto Vincentio wrote them partly in Antwerp (1617-1620) and partly in Louvain (1621-1635). In the references found in the other manuscripts, these two volumes are called, respectively, *Tomus Primus* and *Tomus Secundus*. Dating from the same period in Louvain is a manuscript in Volume 12, folios 216-237. It was written by Guilielmus Boelmans and is entitled *De Progressionibus*.

3. The next manuscripts are the drafts of the treatises sent to Grienberger in Rome. They are to be found in Volume 1 and were written, respectively, by Guilielmus Boelmans (November 1624), Ignatius Derkennis and someone unknown (January 15, 1625), Sancto Vincentio (the beginning of 1625), and Theodorus Moretus (May 22, August 22, September 19, and October 17, 1625).

In Rome (1625-1628) Sancto Vincentio started Volumes 14 and 15, which he later completed in Prague. These volumes are called *Chartae Romanae* in the references of the other manuscripts.

5. Theodorus Moretus came to Prague to rewrite some papers of Sancto Vincentio. Volume 6 contains, in Moretus' hand, *Line-*

arum Potentia, *Planorum Proprietates*, and *Circulorum Proprietates*, and in Volume 12, *Progressio*. In this period Sancto Vincentio composed *De Progressionibus Geometricis*, which is also found in Volume 12.

6. In Ghent (1632-1641) Sancto Vincentio wrote Volumes 10 and 11, which he called *Chartae Gandenses* whenever he referred to them in the other manuscripts.

7. After he obtained the manuscripts rescued from the Prague fires, he began the provisional drafts and even some definitive texts of the *Problema Austriacum*. They can be found in Volumes 1, 2, 6, and 12.

8. Volume 8 was written after 1647 during the controversy over the *Problema Austriacum*.

9. Finally, during the last twenty years of his life, he wrote Volumes 7 and 16, from which, after further rewriting of Volumes 3, 4, 5, and 9, the *Opus Posthumum* originated.

THE MANUSCRIPT OF DE AGUILON (VOLUME 6, FOLIOS 215-315)

The manuscript of de Aguilon is important for two reasons: first, it is the only mathematical manuscript attributed to de Aguilon which is known at this time; and second, it was also a source of inspiration for Sancto Vincentio.

In the preface of his book [1613], de Aguilon promised that after the six books about optics, four books about Catoptrics and several more about Dioptrics would follow. But on March 20, 1617, de Aguilon died and none of the promised books were published. The question remains: are the propositions of this manuscript of de Aguilon a part of the promised books? In this manuscript no proposition can be found which deals directly with optics, but it does consider trisection of the angle, harmonic sets of points or of lines, and the construction of the axes of the ellipse. These topics suggest that the manuscript of de Aguilon was meant as a part of the promised books about optics.

The theory of harmonic sets starts with a definition: if a line AC is divided in three parts by the points D and B so that the whole is to one of the extreme parts as the other extreme part is to the middle, the line is said to be divided *in proportione totius et partium*. In modern language, this definition is equivalent to: the pair A,B is separated harmonically by the pair D,C if $AC/CB = AD/DB$.

De Aguilon constructs harmonic sets of points and of lines and obtains what now could be called the pole and polar line with regard to a circle; namely, given a point A and a straight line through a circle, if all the lines from A intersect the given straight line in B and the circle in C and D, so that the points A and B are separated harmonically by C and D, A is called the pole of the given straight line, and the line in question is the polar line of A with regard to the circle.

The theory of harmonic sets is appropriate for the study of concave mirrors, about which de Aguilon probably knew from the *Perspectiva* of Alhazen (965-1039) and Witelo (1230-1280). Alhazen knew that in the case of the spherical mirror the incident ray and the reflected ray are separated harmonically by the tangent and the normal at the point of reflection. From this proposition of the *Perspectiva* we can conclude that it was very likely that these propositions about harmonic sets were meant for the further development of the books of catoptrics.

The main proposition concerning poles and axes of the ellipse contains the construction of the axes of the ellipse if a set of conjugate diameters is given. This is the so-called problem of Charles (1793-1880). De Aguilon remarks that he needs this construction because he wants to generalize the theory of his *Opticorum libri sex.* He says that he is going to deal with the cylindrical, the conical, and the spherical mirrors (Vol. 6, folio 294^{v}). In short, it is clear that de Aguilon means to continue his work about optics.

THE MANUSCRIPTS OF 1617-1625

During his stay in Antwerp, Sancto Vincentio investigated the trisection of the angle and the finding of two mean proportionals, the properties of conic sections, and the method of exhaustion. In Louvain he wrote about *ducere planum in planum* (see the Appendix) and the *ungula*, and he prepared the papers to convince Grienberger that he could handle the squaring of the circle.

Sancto Vincentio's attempt to trisect the angle is found in the manuscript entitled *Sectio Angulorum.* One can detect the sources used from the note added to the title: "De hac materia agunt Vieta, Clavius, Pappus, Aguillonius." The last name is a reference to the manuscript of de Aguilon. Sancto Vincentio even repeats several trisections from the manuscript of de Aguilon.

Sancto Vincentio also considered another classic problem: the determination of two mean proportionals between two given quantities. He even executed seventeen constructions which were mostly inspired by the *Collectio Mathematica* of Pappos.

Before summarizing the study of the conic sections, we have to cite an important and quite separate proposition, which Sancto Vincentio wrote down apparently without reason (Vol. 13, Proposition 479):

> *When we take away the half AC of AB, from the remainder we take again the half DB, from what remains, DC, we take again the half CE, from the remainder ED again the half DF, from EF again the half EG and so further on, then one can say that the end of this series will be there where AB is trisected.* (Fig. 1)

Figure 1

Sancto Vincentio gave no proof, but said that this proposition can be used for the trisection of the angle. Indeed, what Sancto Vincentio claimed is equivalent to the summation of the series, $1 - 1/2 + 1/4 - 1/8 + \ldots = 2/3$.

In connection with his search for the two mean proportionals, Sancto Vincentio arrived at the study of conic sections. In particular, Volume 17 was meant for this study, but apart from the titles and a few propositions of little importance, nothing of real significance was achieved.

Volume 13, however, contains some important results. Here Sancto Vincentio uses infinite series and conic sections to find the two mean proportionals. In Proposition 673 he claims that the segments of the hyperbola *AXYK*, *KYZI*, ... are equal if *DF*, *DU*, *DZ*, *DY*, *DX*, ... are in continued proportion (Fig. 2).

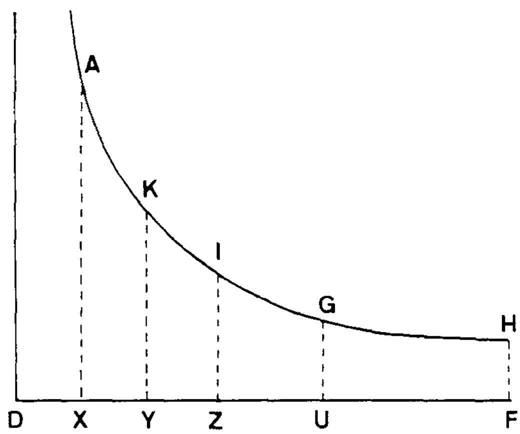

Figure 2

Sancto Vincentio obtained the basic knowledge for this statement from Book III of the *Conics* of Apollonius; he says: Primae propositiones libri tertii Apollonii optime hic conveniunt et par eas segmentorum aequalitas demonstrari potest in triangulis et quadrilateris de quibus ipse agit." ("The first propositions of the third book of Apollonius are very adapt to this situation, and the equality of segments can be demonstrated by them with the help of the triangles and the quadrangles which are treated in that book.")

There is no proof of the above-mentioned proposition which gives, for the first time in the history of mathematics, the logarithmic property of the hyperbola; because of the reference to the work of Apollonius, it is probable that Sancto Vincentio derived this property from the properties of the inscribed rectangles in the hyperbolic segments.

In Proposition 696 Sancto Vincentio derives an important result: if one finds the mean proportional between two given lines, and then the mean proportional between the second line and the third which has just been found, then again between the third and the fourth, and if one continues to repeat this construction, then the *terminus* of this series is one of the two mean proportionals between the two given lines. It is important to note that Sancto Vincentio used the logarithmic property of the hyperbola which he had just discovered; in a prefatory remark he wrote: "The *terminus* of this series gives in a hyperbola a point which trisects the arc of the hyperbola." In the last cited proposition about the trisection he asserted that the trisection is obtained by repeated bisection. Thus, with the aid of the logarithmic property, it is possible to obtain two mean proportionals by repeating indefinitely the process for finding one mean proportional. It is also important to note that here Sancto Vincentio uses for the first time the word *terminus* in a specific sense. A definition of the word *terminus* in this sense cannot be found in the manuscripts, only in the *Problema Austriacum* (1647), where he writes: "Terminus progressionis est seriei finis ad quem nulla progressio pertinget, licet in infinitum continuetur, sed quovis intervallo propius ad eum accedere poterit." ("The *terminus* of a progression is the end of the series, which none progression can reach, even not if she is continued in infinity, but which she can approach nearer than a given segment.") So Sancto Vincentio is the first to give explicitly a definition of the concept of limit.

The use of the infinite series leads, finally, to the theory of exhaustion. Sancto Vincentio took Proposition 1 of Book X of the *Elements* of Euclid as a starting point: If one takes more than the half from a quantity, from what is left also more than the half, and if one keeps doing this, one will obtain at last something smaller than the smallest quantity. Sancto Vincentio goes a bit further than Euclid in the conclusion of his proposition: "The quantity will be exhausted." ("Exhaurietur quantitas.")

Upon this proposition Sancto Vincentio lays the foundation for his calculus: If one subtracts from three given quantities more than their respective half, so that the three subtracted parts are in continued proportion, then by subtracting from the three remaining parts more than half so that the three subtracted parts form the same continued proportion, and by continuing this procedure, the three given quantities are also in the same continued proportion. It is an important proposition because from the ratio of the parts one can deduce the ratio of the wholes. Henceforth this proposition will be called *the* proposition of exhaustion.

Thus we have seen how the mathematical thinking of Sancto Vincentio underwent a clear evolution during his stay in Antwerp. Starting from the problem of the trisection of the angle and the determination of the two mean proportionals, he made use of infinite series, the logarithmic property of the hyperbola, limits, and the related method of exhaustion. Sancto Vicentio later applied this last method, in particular to his theory *ducere planum in planum*, which he developed in Louvain in the years 1621-1624.

In the part of Volume 13 which was written in Louvain, one of the typical theories of Sancto Vincentio occurs for the first time; he called this theory *ducere planum in planum.* He handled this *ducere* quite readily in this manuscript, but although he and his pupils appear to have been accustomed to applying it, no systematic explanation was given. Consequently, the three essential definitions from the printed work, *Problema Austriacum,* are given in the Appendix.

In the manuscript Sancto Vincentio investigates solids which arise by *ductus,* and he is interested especially in the *ductus in se* of half a circle, which consists of two similar parts of a cylinder. He calls these parts *ungulae cylindricae.* In Fig. 3 the two equal ungulae *ABCD* and *ABC'D'* may be seen.

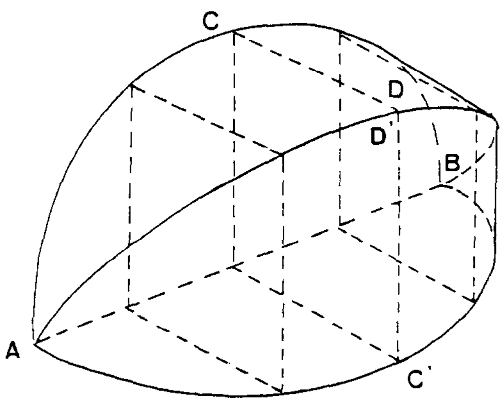

Figure 3

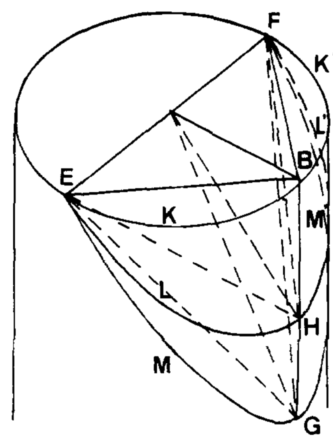

Figure 4

For such an ungula cylindrica and its parts he calculates the volume, using the method of exhaustion. For instance, he says that the volumes of the ungulae *EFBH* and *EFBG* are in the same ratio as their heights *BH* and *BG* (Fig. 4):

> *Pyramids EFBH and EFBG are to each other as BH to BG. The volumes of these pyramids are more than the half of the corresponding ungula. In the remaining parts of the two ungulae more pyramids can be inscribed, two for the ungula EFBH, namely EBKL and FBK'L', and two for the ungula EFBG, namely EBKM and FBK'M', so that the two first if taken together are greater in volume than the half of the remaining part of the first ungula, and the second two if taken together are greater in volume than the half of the remaining part of the second ungula. In addition, the sums of the volumes of the two pyramids are also in the same ratio to each other as BH and BG. This construction may be repeated so that* the *proposition of exhaustion can be used, consequently the wholes, the ungulae, are to each other as BH to BG.*

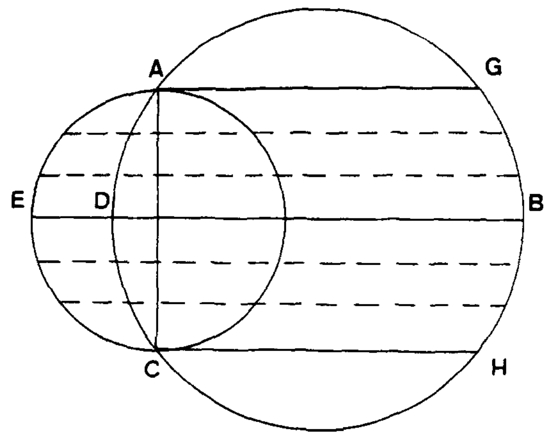

Figure 5

Sancto Vincentio used this method also for many other ductus-figures, too many to describe in this summary. However, he did prove in the same way that the *ductus in se* of half a circle *ACE* is equal to the *ductus* of *ADC* in *AGBHC* (Fig. 5). In the proof the parallel lines to *EB* are very important, so a thorough knowledge of these lines, the ordinates, is needed. Sancto Vincentio even creates a method for the transformation of curves by the translation of these lines. For instance, he says that if one draws on the side *BC* of a triangle, right-angled or not, half a circle, and if one draws *DE* parallel to the base, and if one takes *GE* equal to *FD*, then the points *E* form an ellipse (Volume 17, Proposition 6; see Fig. 6).

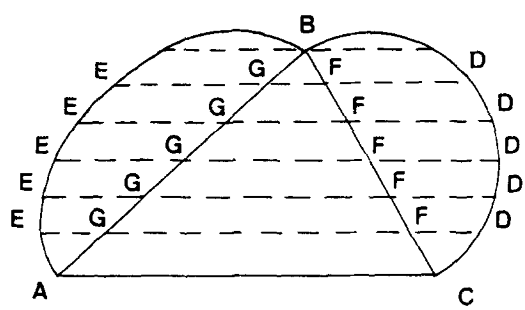

Figure 6

Sancto Vincentio compares many ductus-figures in the manuscripts. So it is easy for him to prove that the *ductus of ABC in se* is equal to the ductus of *AKC* in *ACG* (with $AK = AC$ in Fig. 7), because $AE \cdot EC = DE^2$ or $EI \cdot EF = DE^2$ or $EI \cdot EF \cdot EE = DE^2 \cdot EE$; this is nothing but an equality between the corresponding parallelepipeds which exhaust the two ductus-figures.

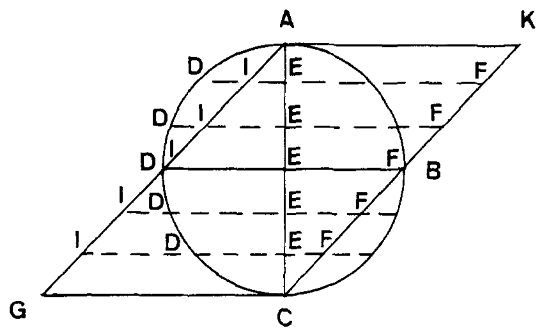

Figure 7

Here Santo Vincentio succeeds in transforming a solid generated from the segments of a circle into the *ductus in se subalterne* of a triangle. Probably he concluded from this that he was able to handle the squaring of the circle, and therefore decided to ask permission of Vitelleschi to publish a work about it.

From the preceding example it is also clear that there is a major difference between the methods of Sancto Vincentio and of Cavalieri: at first sight it seems that both used "lines," but Sancto Vincentio always added a proof by exhaustion to an eventual use of lines. At the same time, it should be emphasized that Sancto Vincentio did not plagiarize Cavalieri's *Geometrica Indivisibilibus continuorum nova*. This plagiarism was suggested by M. Mersenne [1647] after the publication of the *Problema Austricum*.

To convince "judge" Grienberger of the propriety of his method for squaring the circle, Sancto Vincentio and his pupils rewrote the foregoing manuscripts thematically. Guilielmus Boelmans started this work, but his manuscript was not sent to Rome. This manuscript contains *the* proposition of exhaustion and some propositions about infinite geometrical series (Vol. 12, folios 216-237).

In all, there were five consignments sent to Rome. We give only a short summary of the manuscripts found in Volume 1, because they are simply transcriptions of the earlier manuscripts:

1. In a paper of November 1624, the ungula and its parts are discussed, as well as the *ductus obliquus*.

2. The consignment of January 15, 1625, contains *the* proposition of exhaustion, the convergence of geometrical series, the volume of the ungula and its parts.

3. In the beginning of 1625 the *ductus* of segments of a circle is treated.

4. On May 22, 1625, the volume of several ductus-figures which have some connection with the ungula is calculated.

5. The consignment of August 22, September 19, and October

17, 1625, contain lemmata about elementary geometry and propositions of the *ungula elliptica*. This last manuscript is a transcription of the manuscript of January 15 in which the common, cylindrical ungula is treated.

In conclusion, although Sancto Vincentio had made some interesting discoveries, there was no indication that he could handle the squaring of the circle. Consequently Grienberger refused permission for publication.

THE MANUSCRIPTS OF 1626-1631

The Chartae Romanae

In the Chartae Romanae Sancto Vincentio tried other ways to square the circle. In connection with this he studied the *ungula elliptica,* the *ungula parabolica,* and figures of revolution. While studying the latter he found a second method of *ductus,* namely, *ductus in orbem.* This *ductus in orbem* (which he did not publish in the *Problema Austriacum*) is based upon an analogy between rotation and translation. If the semicircle revolves round *AB* it describes a sphere, and if the semicircle is moved in a perpendicular direction to itself it describes, above the plane *ABD,* an *ungula cylindrica.* If one takes for *CD* the diameter of the semicircle, then every formula for a part of the sphere can be applied to a part of the ungula, if π is deleted in the first formula (Fig. 8). Santo Vincentio applied this analogy to the revolution of other figures as well, such as the full circle, the half circle with a rectangle, etc. Starting from the solids acquired by translation he was able to formulate the analogous properties for the solids of revolution.

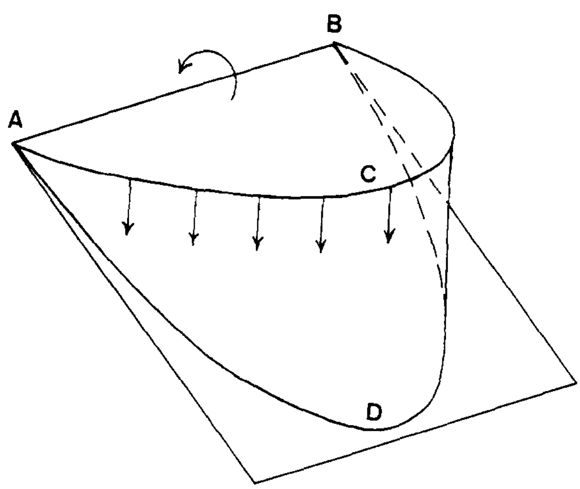

Figure 8

Sancto Vincentio also wrote many propositions about transformations. In this way he arrived at the creation of the virtual parabola ($y^2 = ax^2 - x^4$) and found an analogy between the spiral and the parabola.

Finally, he came to the problem which involves the unsuccessful attempt to square the circle, found in the *Problema Austriacum*.

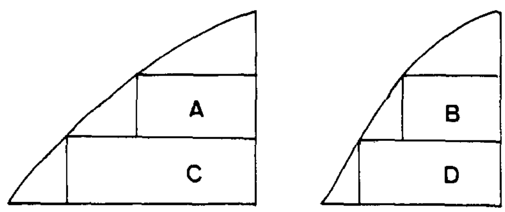

Figure 9

This problem can be put in its most simple form as follows (Fig. 9): In order to find the ratio of the areas of the segments of both figures, one places rectangles in the figures--for simplicity, only two rectangles in each segment are shown. Supposing that the ratios A/B and C/D are known, one wants to determine the ratio $(A + C)/(B + D)$, or as Sancto Vincentio says: How do I have to add (*addere*) the ratios A/B and C/D to get the ratio $(A + C)/(B + D)$? Or, written more symbolically: What is the meaning of the "addition" $*$ in $(A/B) * (C/D) = (A + C)/(B + D)$? Sancto Vincentio was unable to find an answer to this; in addition, he was stricken by an attack of apoplexy, and he had to ask Theodorus Moretus as his assistant to continue his work.

The Manuscripts of Prague

The manuscripts which were written in Prague between Sancto Vincentio's attack of apoplexy and the sack of the city are all thematic transcriptions of earlier manuscripts. The subjects treated are: properties about circles, continued proportions and geometrical series, the harmonic proportion, and properties of lines in triangles.

Sancto Vincentio had to abandon his manuscripts during the sack of Prague; but, luckily, some were saved by Father de Arriaga and taken to Vienna where they remained until 1641.

THE MANUSCRIPTS OF 1632-1641

The Chartae Gandenses

In 1632 Sancto Vincentio arrived in Ghent. During the first nine years of his stay there he wrote Volumes 10 and 11, the so-called *Chartae Gandenses*. In these volumes mainly two kinds of propositions can be distinguished. The first type were mainly reconstructions of propositions taken from the manuscripts lost during the sack of Prague. Probably Sancto Vincentio did not know that the Prague manuscripts had been saved. In the second type he again treated the problem of finding the ratio $(A + C)/(B + D)$ if the ratios A/B and C/D were given. He did not find a solution to this "*additio*," with the consequence that the squaring of the circle was not carried out in the *Problema Austriacum*.

The Texts for the Problema Austriacum

When Sancto Vincentio obtained his salvaged manuscripts in 1641, he wanted to publish them and wrote provisional and final texts for the future *Problema Austriacum*. Volume 1 contains the provisional texts for Book VII. The texts for Books IV, V, and VI are to be found in Volume 2; Volume 6 contains Books I and III, and Volume 12 contains Books II and VIII. It is remarkable that there are no texts for Books IX and X of the *Problema Austriacum*, especially of Book X, which contains the squarings of the circle.

THE CONFLICT ABOUT THE *PROBLEMA AUSTRIACUM*

A short time after publication of the *Problema Austriacum* an attack was published against Sancto Vincentio's squaring of the circle. Marin Mersenee [1647] wrote a short, one-page critique. His argument was twofold: Bonaventura Cavalieri had used the method of *indivisibilia* before Sancto Vincentio (in other words, Mersenne insinuates plagiarism) and Sancto Vincentio had not given a solution to the problem of squaring of the circle, but had only made of it a more difficult problem: namely, given three rational or irrational quantities with the logarithms of two of them, find the logarithm of the third quantity geometrically.

Father A. A. de Sarasa [1649] answered the critique of Mersenne. In part I of his book he solved the problem just mentioned by Mersenne. In the manuscript Volume 8, folios 40-48, *Proaemium*, one can find where Sancto Vincentio helped de Sarasa in composing Part I of his book. In Part II de Sarasa examined Propositions 5, 6, 7, 8, 12, and 54 of Book X of the *Problema Austriacum*, where the "*additio*" of ratios is discussed (the "*additio*" mentioned in connection with Fig. 9). De Sarasa tried to defend this "*additio*," probably as an answer to the objections made by his colleague Ignatius Derkennis. In Volume 8, folios 34-39, under the title *Responsum ad dubium P. Derkennis*, there is a letter of Sancto Vincentio (dated July 3, 1648), probably addressed to de Sarasa. In this letter Sancto Vincentio tries to refute the doubts of his former pupil Derkennis. De Sarasa used this letter when he wrote Part II of his book.

Christian Huygens [1651] claimed that the error in squaring the circle occurs in Proposition 39 of Book X of the *Problema Austriacum* (Oeuvres Complètes de C. Huygens, XI, p. 316). This is a proposition about the "*additio*" of ratios. In Volume 8, folios 58-70, under the title *Liber responsorum geometricorum*, Sancto Vincentio attempted to answer the remark of Huygens. However, the response was restricted to a *Proaemium* (Preface).

The Pole, Alexis Sylvius [1651], wrote a third attack against the squaring of the circle. He produced nearly the same objections as Huygens.

G. A. Kinner de Löwenthurn [1653], a colleague of Sancto Vincentio, tried to defend the impossible "*additio*." Examples of the help Sancto Vincentio gave to Kinner may be seen in the following manuscripts:

Quadraturae explicatio (Vol. 8, folios 100-101).

the draft of a letter of February 22, 1652, addressed to someone in Prague (Vol. 8, folios 102-103). An analysis of the contents indicates that this person was Kinner, for there is almost literally a proposition due to the work of Kinner.

the draft of another long letter: "Misi hoc exemplar Pragas duplici custode, primam partem misi 5 Julij, secundam 12 eiusdem 1652." In this letter Sancto Vincentio discusses the concrete remarks which were made by Huygens (Vol. 8, folios 104-107).

From these three manuscripts it seems clear that Sancto Vincentio was the inspiration behind Kinner's book, and that Kinner had simply recorded the ideas of Sancto Vincentio.

A year later Vincent Leotaud [1654] published a new attack against the squaring of the circle. Leotaud remarked that the "*additio*," as explained by de Sarasa, was completely meaningless.

Meanwhile another attack was spread by the French astronomer Adrien Auzout in the manuscript *Tractatus de rationibus*. There is no copy known, but parts can be found in the work of F. X. Aynscom (see below). Somewhat later a work of Marcus Meibom [1655] was published in Copenhagen. These works are very similar in that they only accept the multiplication of ratios and an addition is not allowed.

Eventually a defense by Franciscus Xaverius Aynscom [1656] was published. In Book I Aynscom explained the theory of ratios. He took nearly all of his data from Sancto Vincentio, as can be seen from Volume 8, folios 49-57, 385-403. In Book II he tried other ways to produce the solution of the squaring of the circle, and he attempted to refute all the critiques which had been published against the *Problema Austriacum*. Once again Sancto Vincentio had guided the response from behind the scenes. His help to Aynscom is clear in Volume 8, folios 71-99, 1-33, 108-378. Moreover, Aynscom was at that time in Ghent, so that he could call on Sancto Vincentio whenever he wanted. Sancto Vincentio and Aynscom continued to work on their "*additio*."

Huygens [1656] insisted that neither Aynscom nor Sancto Vincentio had been able to give the proportion between the circumference and the diameter of the circle (Oeuvres complètes de C. Huygens, XII, p. 276). This was met with nothing but silence from the supporters of Sancto Vincentio.

A final attack was published by Vincent Leotaud [1663]. This book contained nothing new, and the debate about the *Problema Austriacum* soon died out.

THE MANUSCRIPTS FOR THE *OPUS POSTHUMUM*

The preliminary texts on the topic of the duplication of the cube are in Volumes 7 and 16, and in Volume 3, folios 86-202, 228-378, 430-583. There is much evidence of revision and rewriting in Volumes 3, 4, 5, and 9. The final texts for the last two books of the *Opus Posthumum ad Mesolabium*, Books IV and V, were written by de Sarasa, probably after the death of Sancto Vincentio. Finally, the manuscripts for the *Opus Posthumum* do not contain new discoveries, nor do they go beyond the study of conics or of ratios.

In Volume 8 there are also two manuscripts which have nothing to do with the two printed works:

Assertio de vera causa aestus marini (Vol. 8, folios 379-382) and *Assertio de aestus maris* (Vol. 8, folios 383-384).

Sancto Vincentio writes that ebb and flow of the seas are caused by a rising and descending movement of the center *A* of the earth through the center *U* of the universe. If the earth moves upward the mass of water tends to keep *U* as center, with the result that there is a tidal flow at the bottom of the earth.

CONCLUSION

The study of these manuscripts reveals that Sancto Vincentio's most creative period fell between 1617 and 1620; during these years he worked on infinite series, discovered the logarithmic property of the hyperbola and, using the concept of limit, developed a method of exhaustion which was more rigorous than the method of *indivisibilia* published by Cavalieri in 1635. About 1620 Sancto Vincentio had no difficulty applying his method *ductus plani in planum*. Finally, between 1625 and 1627 he discovered the analogy between the spiral and the parabola. Again he cannot be accused of having plagiarized the work of Cavalieri, who published this analogy somewhat later in 1635.

The manuscripts also demonstrate that Sancto Vincentio's creativity was dramatically reduced after his first attack of apoplexy. Moreover, he had to devote much energy to the dispute over the *Problema Austriacum*. When at last he decided to attack the duplication of the cube, his creativity was greatly diminished, and de Sarasa had to give the finishing touches to the manuscripts.

APPENDIX: Ductus plani in planum

The first definition. "Ductus plani *ACDB* in planum *EFG*" (Fig. 10). In order to obtain the *ductus* of plane *ACDB* in plane *EFG* one lays the equal lengths on each other and one of the two planes is put perpendicular to the other. Then all possible rectangles *HI.KG* are formed and the solid obtained in this manner is the *ductus* of *ACDB* in *EFG*.

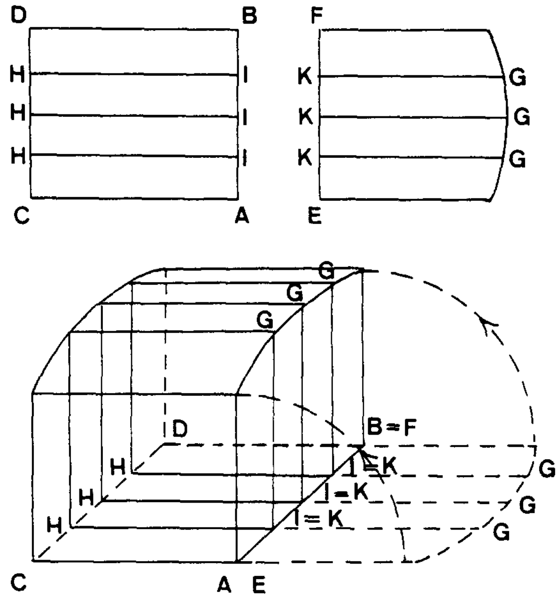

Figure 10

The second definition. "Ductus plani *ABC* in se" (Fig. 11). One takes the figure *ABC* twice, placing them symmetrically with respect to the common altitude *BC*. Then a solid *ABCGH* is constructed, as in the first construction.

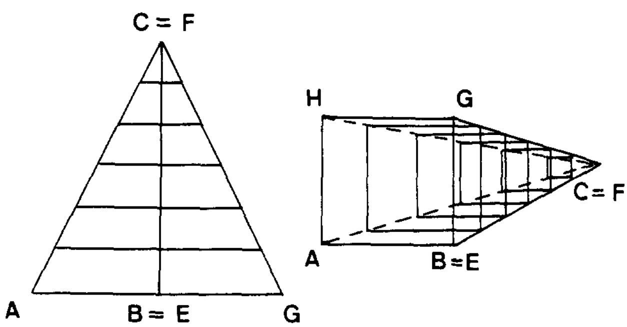

Figure 11

The third definition. "Ductus plani *ABC* in seipsum subalterne." In Fig. 12 the two identical figures are reversed along their common altitude; afterwards one of the two is raised, producing the ductus-figure *ABCG*.

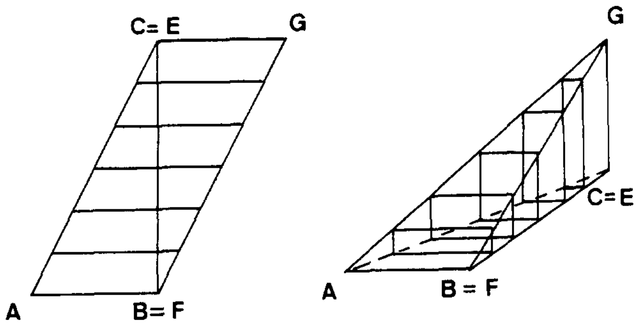

Figure 12

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